INTERNAL RESONANCES AND STABILITY OF ELASTIC SHELLS

V. V. Novikov

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For a number of loaded elastic shells the so-called upper critical load q_u defined in connection with the treatment of the linearized problem exceeds by several times the critical load q_{exp} found in most of the experimental papers; the experiments give a large scatter in q_{exp} (from 0.1 q_u to q_u). These facts are explained at present by the effect of initial inaccuracies in the shape of the shells. Attention should be given to the problem of the stability of shells on the basis of the distinctive features of the spectrum of their eigenfrequencies. In this connection we will point out that a thorough review of the contemporary state of the theory of the distribution of eigenfrequencies of elastic shells is contained in [1]. It is shown in [2] that there exists a class of shell stability problems in which the distribution of the eigenvalues starts from a condensation point, and therefore the normal transition for shell theory from the discussion of a system with an infinite number of degrees of freedom to the study of a system with one degree of freedom is incorrect. Subsequently, a theory [3] for the approximation of such distributed systems by finite-dimensional systems has been constructed.

1. The essential characteristic of the problems under discussion here consists of the fact that at some $q < q_u$ a new stable equilibrium state appears with a lower system energy and that the stability is disrupted in modes having a very large number of bends, i.e., as the load increases, frequencies ω_i with $i = n \sim h^{-1/2}$ (usually $n \sim 10$-100, depending on the shell thickness) undergo the greatest change. The frequency branches of $\omega_i(q)$ intersect with the branches corresponding to $i < n$ as the load increases, but at $q = q_u$ one of them (ω_N) vanishes.

With such a nature for the dependence $\omega_i(q)$ it is necessary to take into account in the equations of elastic vibrations anharmonic effects produced by quadratic nonlinearity, which result in combination vibrations of weak intensity being superimposed on vibrations with frequencies ω_i. In addition starting from some value q_* of the load, such frequency matchings are possible when the conditions of internal resonance

$$\omega_i + \omega_j = \omega_k,\ i + j = k \tag{1.1}$$

are satisfied. The number of points at which they occur can be estimated from the number of coincidences of the frequencies of different modes C_N^2 ($\sim 10^2$-10^3) for the problems under discussion).

The systems under discussion here are subjected to the action of only conservative loads; therefore in the case of a random perturbation, which one can represent in the form of a set of small vibrations in several modes, internal resonance leads to a redistribution of energy among the vibration modes, which can facilitate an increase in the amplitudes of the individual modes to values at which irreversible changes will occur in the shell material and the stability will be disrupted or the system will jump to another equilibrium state. From the point of view of the theory developed in [3], energy redistribution among modes should also be considered as a factor which leads to shell instability. Depending upon the initial conditions, the nature of the loading, and so on, disruption of stability will occur at loads from the range $[q_*, q_u]$.

For practical calculations one can take q_* to be the value of the load at which the intersection of the branches of $\omega_i(q)$ first occurs. It is defined as the maximum value of the load at which $d\omega/di \geqslant 0$ is satisfied for all i. If $d\omega/di < 0$ for some i, then slightly distinguishable frequencies corresponding to adjacent i are possible which satisfy (1.1) together with a small ω_1. It is understood that for an accurate determination of the critical load with internal resonances taken into consideration knowledge of the form of the non-

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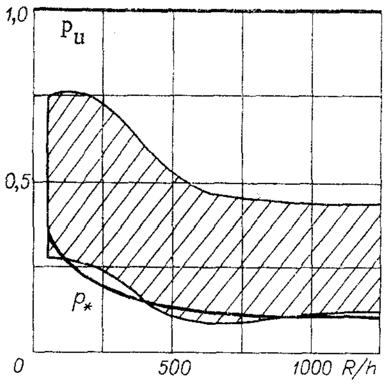

Fig. 1

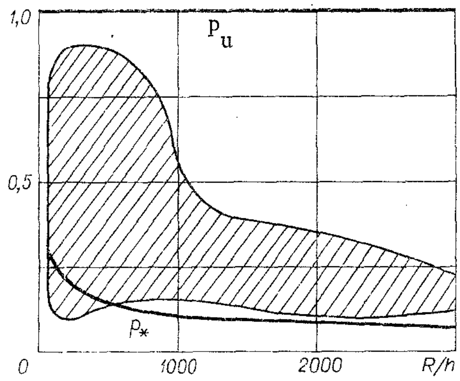

Fig. 2

linearity and the nature of the loading, which is very different in each specific case and leads to a complicated problem, are necessary. Consideration of the spectrum of frequencies of a linearized system permits indicating for all problems of this class the loads for which stability loss is possible.

2. We discuss two examples. We will use the frequency equations for spherical and cylindrical shells [4] in the form of [5] supplemented by terms containing a load, which one can easily derive after taking into account in the equations the elastic vibrations of the forces produced by equilibrium stresses.

A. Spherical shell under hydrostatic pressure p. The frequency equation is of the form

$$\frac{2\rho R^2}{E}\left(1+\frac{1+3\sigma}{k}\right)\omega^2=-\frac{kp}{E}\frac{R}{h}+\frac{k^2}{6(1-\sigma^2)}\left(\frac{h}{R}\right)^2+2,$$

where h is the shell thickness, R is the radius, ρ is the density of the material, σ is the Poisson coefficient, E is the tension modulus, and $k = n(n + 1)$, $n = 2, 3, \ldots$. The well-known fact that stability disruption occurs at $p \sim (h/R)^2$ is used in deriving this relationship.

By setting $\omega^2 = 0$ and minimizing p with respect to k, we have

$$p_u=\frac{2E}{\sqrt{3(1-\sigma^2)}}\left(\frac{h}{R}\right)^2.$$

We will make use of the relationship $d\omega/dn \geqslant 0$, which one can replace by the equivalent inequality $d\omega^2/dk \geqslant 0$, in our search for p_*. From this we have

$$p_*=\left(\frac{3}{2}\right)^{1/3}E\left[\frac{1+3\sigma}{(1-\sigma^2)^2}\right]^{1/3}\left(\frac{h}{R}\right)^{7/3},\quad \frac{p_*}{p_u}=\frac{3}{2}\left[\frac{(1+3\sigma)^2}{12(1-\sigma^2)}\left(\frac{h}{R}\right)^2\right]^{1/6}.$$

Plots of p_u and p_* and the region of experimental data for p_{exp} taken from [5] are shown in Fig. 1; p_u and p_* were calculated with $\sigma = 0.3$.

B. Cylindrical shell compressed in the axial direction by a force p. The frequency equation is of the form

$$\frac{\rho R^2}{a^2}\omega^2\left(1+\frac{1}{a^2}\right)=-p+\frac{E}{a^2}+\frac{E}{12(1-\sigma^2)}a^2\left(\frac{h}{R}\right)^2,$$

where $a = n\pi(R/l)$ $(n = 1, 2, \ldots)$ and l is the shell length.

Similarly to the previous example, we find

$$p_*=E\left[\frac{3}{16(1-\sigma^2)^2}\right]^{1/3}\left(\frac{h}{R}\right)^{4/3},\quad \frac{p_*}{p_u}=\frac{3}{2}\left[\frac{1}{12(1-\sigma^2)}\right]^{1/6}\left(\frac{h}{R}\right)^{1/3}.$$

Plots of p_u and p_* and the region of experimental critical loads p_{exp} [5, 6] are presented in Fig. 2.

We note in conclusion that the load p_* found depends upon the shell thickness and agrees over a wide range of variation of h with the lower limit of the region of the derived and experimental critical loads. Other experimental facts find explanation within the framework of the proposed mechanism of stability disruption. In particular, the large scatter of the experimental data can be explained by a difference in the initial conditions, the nature of the loading, and random dynamical effects, and the decrease of this scatter at small thicknesses is related to an increase in the number of resonance relationships, along with the increase in 1/h.

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FIELD OF ELASTOPLASTIC STRAINS IN THE MOUTH ZONE OF A CRACK

V. I. Arkhipov

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When establishing the criterion of failure, the knowledge of the strain and force conditions in the mouth zone of the crack is of great importance. Dependent on the model of elastoplastic strain of this zone, different concepts are proposed for the choice of the failure criterion [1]. At the same time it is necessary to take into account the fact that a real failure process on the macro scale in the majority of cases has a mixed character [2]. Hence experimental and theoretical investigation of the field of elastoplastic strain (EPS) in the mouth zone of the crack [3] is of great importance. In [4-6] an experimental analysis of the EPS fields in the mouth zone of a crack is carried out by methods of photoelasticity, Moiré and holographic interferometry. In [7-9], by numerical methods, analysis of EPS fields is carried out for plane stress and plane strain states. A paper should be mentioned [10] in which by the holographic interferometry method, the field of elastic and residual components of strains is determined for a plate loaded by internal pressure.

<u>1. The Method of Investigation.</u> The investigation was carried out under the normal external conditions on flat testpieces of alloy steel 38KhNVA. The testpieces were heat treated according to typifying conditions. Hardness HRC = 50.

In Fig. 1 we have shown the geometry of the testpieces. The origin of the coordinate system is connected with the crack tip. The work zone of the testpieces was mechanically polished, and was then lapped under a load $P_0 \sim 2$kN to obtain a maximum flatness and a satisfactory coefficient of light reflection. The testpieces had an initiated fatigue crack. The thickness of testpieces t = 2.0 mm. The loading was carried out on a testing machine provided with a laser holography system. The loading regime was stepped monotonic tension with the load step Δp = 981 H.

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